106385 SEMINAR IN TOPOLOGY 2 - WINTER SEMESTER 2017/18

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A *CAT(0) space* is a geodesic metric space that satisfies a type of non-positive curvature. The Euclidean plane, the Hyperbolic plane, trees, the Davis complex for Coxeter groups, and the geometric realization of any building are all examples of CAT(0) spaces. When Gromov first coined the term CAT(0) in the late 80s he gave criterion for building new CAT(0) spaces by gluing together polyhedra from $\mathbb{E}^n$ and $\mathbb{H}^n$. If the polyhedron are all unit length Euclidean n-cubes, the space obtained is a *CAT(0) cube complex*.

At the time, no one had any idea what a CAT(0) cube complex might be useful for. Now, almost 30 years later, it has become clear that a great many finitely presented groups act on CAT(0) cube complexes, and that such actions can reveal secrets about the group itself.

I will present the basic geometry of CAT(0) cube complexes, show how group actions are constructed in practice, and finally explain what it all means. I hope that those who follow along will come to understand the significance of many recent results in the field of geometric group theory and low dimensional topology, as well as develop intuition and know-how within geometric group theory.

Over the course I hope to cover the following subjects in a range of breadth and depth.

(1) Residual Finiteness and Subgroup Separability,
(2) CAT(0) Spaces and how to build them,
(3) The Geometry of CAT(0) Cube Complexes,
(4) Sageev's Dual Cube Complex Construction,
(5) Virtually Special Cube Complexes,
(6) Canonical Completion and Retraction,
(7) Virtual Specialness of Coxeter Groups and Small Cancellation Groups,
(8) Groups which cannot be cubulated,
(9) Biautomaticity of cocompactly cubulated groups,
(10) The Resolution of the Virtual Haken Conjecture,

Prerequisites: The material should be accessible to those who are familiar with the basic notions in topology, such as fundamental group and covering spaces, the hyperbolic plane, and finitely presented groups (the free group in particular). Knowledge of things like Bass-Serre theory, small cancellation theory, and 3-manifolds should be considered a major asset. I am happy to devote time to filling in the gaps.